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EXPERIMENTAL STUDY OF THE INTERACTION OF A PAIR OF
HYPERSONIC JETS

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Jet interaction in certain types of vacuum pumps appreciably affects the evacuating capacity and the limiting vacuum. However, there have been no goal-oriented studies on this phenomenon applicable to vacuum pumps [1]. Jet interaction studies [2-4] carried out for fairly high Reynolds numbers characterizing the viscous effects, and small values of nozzle spacing do not relate directly to the operating conditions of vacuum pumps. The interaction of a pair of adjacent jets is similar to the interaction of a single jet with a surface parallel to the jet axis, without friction. The formulation of such studies is of interest in solving problems associated with the effects of jet strength on the surrounding elements under the conditions of vacuum. The present paper is devoted to the experimental study of the influence of viscous effects on the density distribution in the symmetry plane of a pair of parallel, strongly underexpanded hypersonic jets under conditions similar to vacuum pumps.

The flow structure in the interaction region of jets issuing into a heated space is determined by Mach number M at the nozzle section, ratio of specific heats γ , stagnation parameters of the fluid jet p_0 and T_0 , pressure in the surrounding medium p_k , outside temperature T_k , Reynolds number Re_* based on the parameters at the throat section, and the geometric parameters: h , the distance between the nozzles, d_* , the throat diameter. In order to determine the location of the geometric surfaces it is sufficient to use the ratio of

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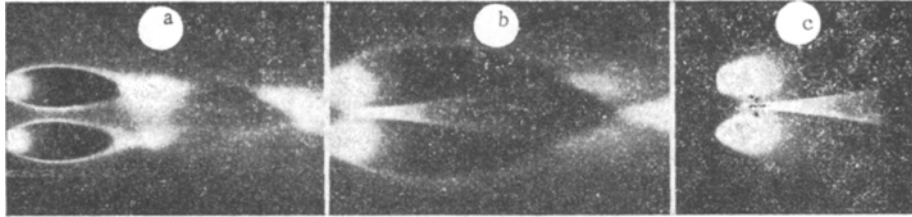


Fig. 1

transverse dimensions of single jet $D \sim d_* \sqrt{p_0/p_k}$ and the distance between the nozzles h . Photographs of the longitudinal section of a pair of interacting jets formed at constant stagnation pressure p_0 are shown in Fig. 1a-c; the pressure in the surrounding medium is a maximum in the case a. Figure 1a refers to the case $D < h$ when the jets do not interact and the structure of each jet does not deform due to the influence of the neighboring jet. When the distance D is of the order of h (Fig. 1b), the ratio D/h appreciably affects the flow geometry and the distribution of parameters in the plane of symmetry. For $D \gg h$ (Fig. 1c) it is possible to expect weak influence of the ratio D/h on the distribution of parameters in the jet interaction region with dimensions $\sim h$. In the present paper the last case is considered when $D \gg h$.

The effects of viscous jet interaction with the surrounding fluid lead to the formation of a mixing layer at the jet boundary and can appreciably affect the distribution of fluid parameters in the jet interaction region. Jet mixing with the surrounding fluid for $M = \text{idem}$ and $\gamma = \text{idem}$ is determined by Reynolds number $Re_L \sim Re_* / \sqrt{p_0/p_k}$ [5, 6]. Here Re_* is the Reynolds number based on parameters at the throat. In the case of a viscous fluid with $p_0/p_k \gg 1$ in the interaction region, viscosity may have an effect in the neighborhood of the point of spreading, and greater its value, more is the rarefaction. For this region the characteristic Reynolds number $Re_h \sim Re_* d/h$ is analogous to the criterion Re_L for the free jet and had been introduced earlier in [3].

At large distances from the nozzle section, the flow in the undisturbed region can be described by a source. Hence the density ahead of the shock that appears with interacting jets has an approximate form

$$\rho \approx (d_*/h)^2 \rho_0 \varphi(M, \gamma, x/h, y/h),$$

where x, y are the longitudinal and transverse coordinates. When $h \gg d_*$ the flow in the interaction region is hypersonic and according to the principle of hypersonic stabilization, flow parameters behind the suspended and reflected waves from the plane of symmetry do not depend on free stream Mach number and are determined as parameters of the source: M, γ , and the distance between the nozzles. Thus, the above relation for density can be put in the form

$$\rho \approx (d_*/h)^2 \rho_0 f(M, \gamma, x/h, y/h, Re_L, Re_h).$$

Analysis of experimental data in the form of the dependence of nondimensional density on the throat section parameters follows from this.

Experiments were conducted in a hydrodynamic setup equipped with cryogenic evacuation and electron beam diagnostics [7]. Jets were created by two identical nozzles with 1 mm diameter at the throat, the nozzle half angle was 15° and the geometric Mach number at the nozzle section was 3. The flow conditions were selected (see Table 1) such that they covered conditions that are characteristic of the jets in vacuum pumps, in particular, optimum in terms of Re_L for the evacuating capacity of the free jet [8].

Photographs of plane jet sections (Fig. 1a-c) as well as the density distributions in the plane of symmetry were obtained using electron beam technique. The transverse density distribution profiles in the plane of symmetry are shown in Fig. 2. The numbers on the curves refer to the type of flow given in Table 1.

The density distribution profile in the axis of symmetry are shown in Fig. 3 for two groups of conditions with constant distances between nozzles. It is seen that with increase in Re_* density increases, and the location of the maximum is shifted in the direction of the plane of the nozzle section.

TABLE 1

Number of flow conditions	p_0, Pa	Re_*	Re_h	Re_L	$\frac{p_0}{p_R}$	$h \cdot 10^4, \text{m}$
1	$0,35 \cdot 10^5$	$5,3 \cdot 10^3$	92,5	5	10^6	5,8
2	$0,70 \cdot 10^5$	$1,1 \cdot 10^4$	182,5	10	10^6	5,8
3	$1,40 \cdot 10^5$	$2,1 \cdot 10^4$	365	20	10^6	5,8
4	$0,83 \cdot 10^5$	$1,3 \cdot 10^4$	92,5	12	10^6	14
5	$1,66 \cdot 10^5$	$2,5 \cdot 10^4$	182,5	24	10^6	14
6	$3,32 \cdot 10^5$	$5,0 \cdot 10^4$	365	48	10^6	14
7	$3,32 \cdot 10^5$	$5,0 \cdot 10^4$	182,5	48	10^6	28
8	$0,70 \cdot 10^5$	$1,1 \cdot 10^4$	182,5	42	10^4	5,8
9	$1,66 \cdot 10^5$	$2,5 \cdot 10^4$	92,5	24	10^6	28

A change in Re_* for the flow of interacting jets in vacuum is equivalent to a change in Re_h . Density distribution in the general case is also subjected to viscous effects (Re_L) in the jet boundary layer, in other words, there is an ejector effect of jets. At lower Reynolds numbers its role should be stronger. Apparently, the overall impact of viscosity inside the jet and on its boundaries can be explained by the shift in the maximum density (and, consequently, even the point of spreading) downstream with the reduction in mass flow and without change in system geometry.

Representing the results of all conditions (see Table 1) in terms of the coordinates $(\rho/\rho_0)(h/d_*)^2 = f(x/h)$ shows a generalization of density distribution in individual sets of conditions:

1. The conditions 7 and 8 with identical Re_L and Re_h (Fig. 4) correlate very well.

2. The pair of conditions 2 and 4 and 3 and 5 (Fig. 4) also correlate well which can also be extended to low Reynolds numbers Re_L . These conditions have close values of Re_L and appreciably different values of Re_h . This means that at low values of Re_L , when there is still no clearly defined shock layer, the critical parameter is Re_L and the effect of Re_h is less or is not so significant, apparently just as for the flow in vacuum.

3. The condition 6 does not correlate with 7 and 8. It has the same Re_L as the last but twice the value of Re_h . It thus follows that sufficiently well-formed compressed layer (at $Re_L \approx 48$) considerably changes the flow geometry along the jet periphery, affecting the density distribution in the region of the maximum, i.e., in this case there is not even a kinematic similarity.

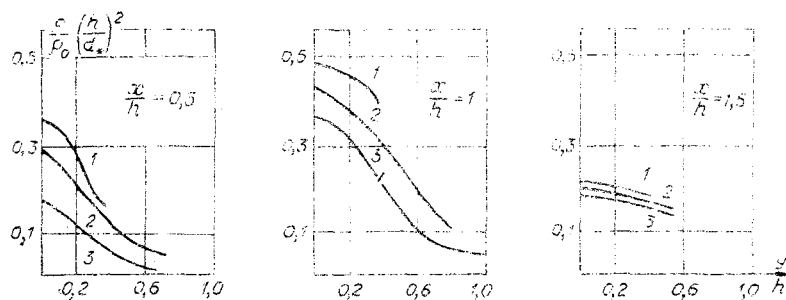


Fig. 2

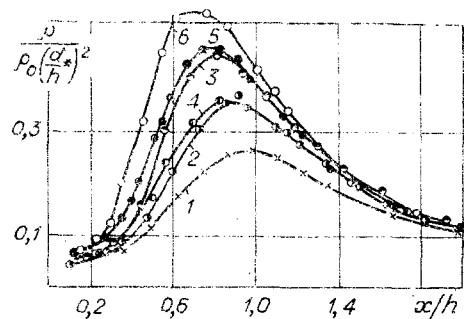


Fig. 3

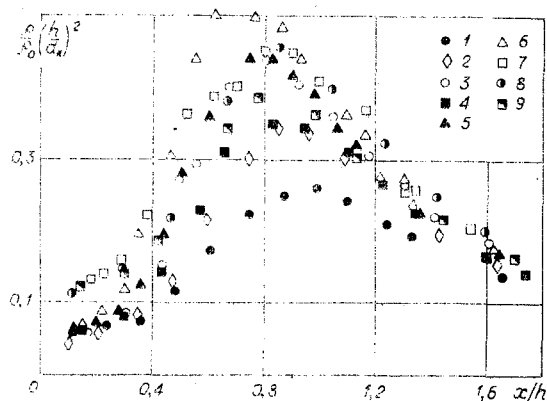


Fig. 4

4. At large distances from sources the density distribution can be expressed by one relation for all the conditions. This means that the effect of the conditions for the formation of the peripheral jet region is local.

There are less significant but noticeable features of density distribution for individual conditions at small distances from the plane of the nozzle section. In conditions 7-9, unlike all other cases, the normalized density at the initial segment has a higher value compared to the generalized relations at $x/h \leq 0.3$. It is possible that this is the result of the formation of reverse flow from the initial segment of the interaction region or the result of ejection (jet surface in these cases is the most well developed).

The absence of a theoretical model for this phenomenon and the limitations in experimental data pose the question of the correctness of such a procedure. Viscous effect in the nozzle on the density distribution in the jet interaction region was not considered.

Estimates of the effects of condensation, using [9], showed that condensation is initiated in the hypersonic region of the source and has weak effect on its parameters.

Thus, conclusions concerning the qualitative impact of viscous effects on the density distribution in the jet interaction region and also the generalization for same values of Reynolds numbers can be considered established. The study of the spreading region, viz., the neighborhood of the stagnation point in the plane of symmetry, remains theoretically interesting and practically important for vacuum jet evacuation systems.

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FLOW IN THE HYPERSONIC BOUNDARY LAYER ON A FINITE TRIANGULAR
WING IN THE PRESENCE OF AN ANGLE OF ATTACK

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The investigation of three-dimensional viscous gas flows at hypersonic flight velocities is of importance to the determination of the aerodynamic characteristics. It has been established in numerous experimental studies (see [1], for instance) that the nature of the flow in the boundary layer on flat delta wings depends substantially on the magnitude of the hypersonic interaction parameter $\chi = M_\infty^2 \text{Re}^{1/2}$, where M_∞ is the free stream Mach number, and $\text{Re} = \rho_\infty U_\infty L / \mu_0$ is the Reynolds number determined from values of the density and velocity in the unperturbed stream, the wing length, and the viscosity coefficient at the stagnation temperature. Two limiting flow regimes can be examined here. In the weak interaction regime ($\chi \sim 0.1$) even for a small angle of attack vortices [2] which drift downstream, occur within the boundary layer on the leeward side of a delta wing, and their interaction with the body surface results in an increase in friction and heat flux. In the strong viscous interaction regime ($\chi \geq 1$) [3], at least up to moderate angles of attack, attached flow is realized over the whole wing. However, it should be noted that the nature of the flow in a region near the apex of the wing is identical in both cases since the parameter is $\chi \geq 1$ there (the Reynolds number should be calculated relative to the length of the domain under consideration). The flow around a thin delta plate in the strong viscous interaction regime has been investigated theoretically at zero angle of attack in [4-7]. Examination of the flow around a semiinfinite triangular plate permits reduction of the boundary value problem to a self-similar problem, for whose solution the methods developed for two-dimensional problems are applicable. However, the system remains three-dimensional in the consideration of the flow around a delta wing at an angle of attack in the strong viscous interaction regime. A solution is obtained in [8] for the system of Navier-Stokes equations near a semiinfinite delta wing at an angle of attack, but an assumption is made here that the gradients in the radial direction are much less than in the others, and the boundary-value problem is reduced to a self-similar problem.

1. The flow of a hypersonic stream of viscous gas around a finite delta wing at an angle of attack α° is considered in this paper under the assumption that the perturbed part of the flow contains a stream inviscid in a first approximation, which is described by the hypersonic theory of small perturbations [9] and the viscous boundary layer. It is assumed that the angle of attack is small ($\alpha^\circ < \tau$) and such that the assumption of the hypersonic theory of small perturbations is always satisfied

$$M_\infty(\tau \pm \alpha^\circ) \geq O(1), \quad (1.1)$$

where $\tau = (s/\text{Re})^{1/4}$ is the characteristic dimensionless boundary-layer thickness ($s = \tan \beta$, β is the half-angle at the wing apex). The plus sign in (1.1) corresponds to flow around the lower (windward) wing surface, and the minus sign around the upper (leeward) surface. The Cartesian coordinate system whose origin is at the apex of the delta wing (the x° axis is directed along the axis of symmetry, the z° axis along the span, and the y° axis along the normal to the wing surface) is presented in Fig. 1. It is assumed that boundary-layer interaction with the external hypersonic stream is strong ($\chi > 1$) on the whole wing. The

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